ZIMBABWE

MINISTRY OF PRIMARY AND SECONDARY EDUCATION

Pure Mathematics Non-Formal Syllabus

LEVEL 3

Curriculum Development Unit

P. O. Box MP 133

Mount Pleasant

HARARE

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1.0 PREAMBLE

1.1 INTRODUCTION

In developing the compressed Pure Mathematics Level 2 syllabus attention was paid to the need to provide continuity of pure mathematical concepts and lay foundations for further studies. It is meant for learners who have the ability and interest in studying Pure Mathematics. The learning phase will provide learners with opportunities to apply pure mathematical concepts, principles and skills in other learning areas.

The intention is to provide wider opportunities for learners who wish to acquire competences in scientifically and technologically based areas required for the national human capital development needs and enterprising activities in the 21st century. In learning Pure Mathematics, learners should be helped to acquire a variety of skills, knowledge and processes, and develop positive attitude towards the learning area. These will enhance the ability to investigate and interpret numerical and spatial relationships as well as patterns that exist in mathematics and in the world in general. The syllabus also caters for learners with diverse needs to experience Pure Mathematics as relevant and worthwhile. It also desires to produce a learner with the ability to communicate mathematical ideas and information effectively.

1.2 RATIONALE

Pure Mathematics is the science of abstract concepts in the context of number, time, and space. It uses logic and reasoning to construct new mathematical structures by building on existing ones, and to discover new patterns within, and relationships among mathematical structures. Pure Mathematics is underpinned in abstraction and proof. Typically, the Pure Mathematician finds joy and fulfillment in the intrinsic value of her/his constructions and discoveries, with little or no regard to their applications in life. However, examples abound of instances where Pure Mathematics discoveries initially born and existing only in the abstract, have eventually been found to have important applications in life. These life examples are in the areas of science, technology and engineering, as well as finance, banking and commerce, among others. Given the Pure Mathematician's interest and ability in abstraction and pattern discovery, such a professional is increasingly in demand in such activities as meteorology, investment

forecasting, and risk analysis. Thus teaching and learning Pure Mathematics should not be construed as a luxury, but a 21st Century necessity.

The teaching and learning process in compressed Pure Mathematics at Level 3 is also expected to enhance learners' confidence and sense of self fulfillment, interconnectedness and intellectual honesty, hence contributing to their growth in the acquisition of Unhu/ Ubuntu /Vumunhu values.

1.3 SUMMARY OF CONTENT

The syllabus will cover the theoretical and practical aspects of compressed Pure Mathematics. The learning area will cover: algebra, geometry and calculus.

1.4 ASSUMPTIONS

The syllabus assumes that the learner has:

1.4.1 passed at least one of the following at Level 2: Mathematics, Pure Mathematics and Additional Mathematics
1.4.2 interest in studying Pure Mathematics Level 3

1.5 CROSS CUTTING THEMES

The following are some of the cross cutting themes in the teaching and learning of compressed Pure Mathematics: -

1.5.1 Business and financial literacy
1.5.2 Disaster and risk management
1.5.3 Collaboration
1.5.4 Environmental issues

1.5.5 Enterprise skills
1.5.6 HIV and AIDS
1.5.7 Unhu/Ubuntu/ Vumunhu
1.5.8 ICT
1.5.9 Gender

2.0 PRESENTATION OF SYLLABUS

The compressed Pure Mathematics syllabus is a document covering Level 3. It contains the preamble, aims, syllabus objectives, methodology and time allocation, syllabus topics, scope and sequence, competency matrix and assessment. The syllabus also suggests a list of resources that could be used during learning and teaching process.

3.0 AIMS

This syllabus is intended to provide a guideline for level 3 learners which will enable them to:

3.1 develop the abilities to reason logically, to communicate mathematically, and to learn co-operatively and independently
3.2 acquire enterprising skills through modelling, research and project based learning
3.3 develop an appreciation of the applicability, creativity and power of pure mathematics in solving a broad range of problems
3.4 understand the nature of Pure Mathematics and its relationship to other branches of mathematics and STEM in general.
3.5 appreciate the use of I.C.T tools in solving pure mathematical problems
3.6 engage, persevere, collaborate and show intellectual honesty in performing tasks in Pure Mathematics, in the spirit of Unhu/ Ubuntu/Vumunhu

4.0 SYLLABUS OBJECTIVES

By the end of the learning period, the learners should be able to:

4.1 make use of a variety of mathematical skills (including graphing, proving, modelling, finding pattern and problem solving) in the learning and application of compressed Pure Mathematics.

4.2 communicate pure mathematical ideas and information
4.3 produce imaginative and creative work arising from pure mathematical ideas
4.4 choose strategies to construct arguments and proofs in both concrete and abstract settings
4.5 construct and use mathematical models in solving problems in life
4.6 demonstrate perseverance, diligence, cooperation and intellectual honesty.
4.7 use I.C.T tools to solve pure mathematical problems
4.8 conduct research projects including those related to enterprise

5.0 METHODOLOGY AND TIME ALLOCATION

5.1 METHODOLOGY

A constructivist based teaching and learning approach is recommended for the level 3 compressed Pure Mathematics Syllabus. The theoretical basis for this approach is that: in a conducive environment with appropriate stimuli, learners' capacity to build on their pre-requisite knowledge and create new mathematical knowledge is enhanced. A conducive environment in this context is one that encourages: creativity and originality; a free exchange of ideas and information; inclusivity and respect for each other's' views, regardless of personal circumstances (in terms of, for example: gender, appearance, disability and religious beliefs); collaboration and cooperation; intellectual honesty; diligence and persistence; and Unhu/ Ubuntu /Vumunhu. This is particularly important in a learning area like mathematics, given the negative attitudes associated with its teaching and learning.

Providing appropriate stimuli has to do with posing relevant challenges that excite learners, and help to make learning Pure Mathematics an enjoyable, fulfilling experience. Such challenges could be posed in the form of problems that encourage learners to create new (to them) mathematical knowledge/ideas in line with the teacher expectations and even beyond. New knowledge acquired in such a manner tends to be deep rooted and meaningful to learners, hence enhancing their ability to apply it within the learning area and in life. Definitely spoon feeding is not and cannot be an appropriate stimulus, as it does not help learners to develop critical thinking, creativity, and the ability to think outside the box, which are critical for self-reliance, national sustainable development and global competitiveness. Thus learners need to be active participants and decision makers in the pure mathematics teaching and learning process, with the teacher playing a facilitator role.

Pre-requisite knowledge and skills refers to what the learners should already know and can do, which can form a strong basis on which to construct the expected new knowledge. Thus the Pure Mathematics teacher needs to carefully analyse the new concepts

and principles she/he intends to introduce, identify the relevant pre-requisite knowledge, assess to identify any gaps, and take appropriate steps to fill such gaps.

The following, is a list of teaching and learning methods that are consistent with, and supportive of the above approach:

5.1.1 Guided discovery
5.1.2 Group work
5.1.3 Interactive e-learning
5.1.4 Problem solving
5.1.5 Discussion
5.1.6 Modelling

5.2 TIME ALLOCATION

The learning area should be allocated at least two hours per week for the adequate coverage of the syllabus.

Learners are expected to participate in the following activities: -

- Mathematics Olympiads
- Mathematics and Science exhibitions
- Mathematics seminars

6.0 TOPICS

The following topics will be covered at Level 3.

6.1 Algebra
6.2 Geometry and vectors
6.3 Series and Sequences
6.4 Trigonometry

6.5 Calculus
6.6 Numerical methods
6.7 Complex numbers
6.8 Matrices

7.0 SCOPE AND SEQUENCE

TOPIC	LEVEL 3
Algebra	• Indices and proportionality • Polynomials • Identities, Equations and Inequalities • Functions • Relations • Matrices • Mathematical Induction • Groups

TOPIC	LEVEL 3
Geometry and vectors	• Graphs and Coordinate geometry • Vectors (up to three dimensions)

TOPIC	LEVEL 3
Series and Sequences	• Sequences • Series

TOPIC	LEVEL 3
Trigonometry	• Plane Trigonometry • Trigonometrical Functions

TOPIC	LEVEL 3
Differentiation	• Differentiation • Integration • 1st Order Differential equations

TOPIC	LEVEL 3
Numerical Methods	• Errors • Iterative methods • Newton – Raphson method Trapezium rule

TOPIC	LEVEL 3
Complex Numbers	• Equations (up to order 5) • Polar form $(r(\cos\theta + i\sin\theta) = re^{i\theta})$

	• Loci • deMoivre's Theorem • n^{th} roots of unit
SUB TOPIC	**LEVEL 3**
Matrices	• Matrices • Mathematical Induction • Groups

LEVEL THREE (3)

8.0 COMPETENCY MATRIX: LEVEL 3 SYLLABUS

TOPIC	OBJECTIVES Learners should be able to:	CONTENT: {Skills, Knowledge, Attitudes}	SUGGESTED NOTES AND ACTIVITIES	SUGGESTED RESOURCES
Algebra	• Solve problems in algebra	• Indices and proportionality • Polynomials • Identities, Equations and Inequalities • Functions • Relations	• Applying laws of indices to solve problems (including life problems) • Modelling situations involving variation and solving related problems • Solving problems in algebra	• ICT tools • Braille materials and equipment • Talking books or software

				• Relevant texts

TOPIC	OBJECTIVES Learners should be able to:	CONTENT: {Skills, Knowledge, Attitudes}	SUGGESTED NOTES AND ACTIVITIES	SUGGESTED RESOURCES
Geometry and vectors	• locate solutions of equations and inequalities using sketches of graphs • solve problems in algebra involving: - parallel and perpendicular straight lines - distance between two points - reducing an equation to appropriate linear form in solving problems (such as $y = ax^2 + b$ when y is plotted against x^2) - equation of a circle - parametric equations • manipulate vectors in 3 dimension	• Graphs and Coordinate geometry • Vectors (up to three dimensions) • Vector equation of a straight line • Equation of a plane • Cross product	• locating solutions of equations and inequalities using sketches of graphs • solving problems in algebra involving: - parallel and perpendicular straight lines - distance between two points - reducing an equation to appropriate linear form in solving problems (such as $y = ax^2 + b$ when y is plotted against x^2) - equation of a circle - parametric equations • manipulating vectors in 3 dimension	• ICT tools • Braille materials and equipment • Talking books or software • Relevant texts

TOPIC	OBJECTIVES Learners should be able to:	CONTENT: (Skills, Knowledge, Attitudes)	SUGGESTED NOTES AND ACTIVITIES	SUGGESTED RESOURCES
Series and sequences	• use sequence definitions such as $U_n = n^2$ and $U_{n+1} = 2U_n$ to calculate successive terms • find the general terms and other terms of arithmetic and geometric progressions • use standard results for Σr, Σr^2 and Σr^3 to find related sums • use the method of differences to obtain the sum of finite series • solve problems involving - sequences - method of differences as well as Taylor's and McLaurin's series	• Sequences • Series • Standard results • Method of differences • McLaurin's series • Taylor's series	• using sequence definitions such as $U_n = n^2$ and $U_{n+1} = 2U_n$ to calculate successive terms • finding the general terms and other terms of arithmetic and geometric progressions • using standard results for Σr, Σr^2 and Σr^3 to find related sums • using the method of differences to obtain the sum of finite series • solving problems involving - sequences - method of differences as well as Taylor's and McLaurin's series	• ICT tools • Braille materials and equipment • Talking books or software • Relevant texts

TOPIC	OBJECTIVES Learners should be able to:	CONTENT: {Skills, Knowledge, Attitudes}	SUGGESTED NOTES AND ACTIVITIES	SUGGESTED RESOURCES

TOPIC	OBJECTIVES Learners should be able to:	CONTENT: {Skills, Knowledge, Attitudes}	SUGGESTED NOTES AND ACTIVITIES	SUGGESTED RESOURCES
Trigonometry	• Compute calculations involving degrees and radians • solve problems involving lengths of arcs, areas of sectors and segments • use small angle approximation for sin x, cos x and tan x • sketch the graphs of trigonometrical functions • solve problems using trigonometrical identities	• Plane Trigonometry • Trigonometrical Functions • Graphs of Trigonometrical functions • Trigonometrical equations • Trigonometrical identities (excluding half angle identities)	• Computing calculations involving degrees and radians • solving problems involving lengths of arcs, areas of sectors and segments • using small angle approximation for sin x, cos x and tan x • sketching the graphs of trigonometrical functions solve problems using trigonometrical identities	• ICT tools • Braille materials and equipment • Talking books or software • Relevant texts

TOPIC	OBJECTIVES Learners should be able to:	CONTENT: {Skills, Knowledge, Attitudes}	SUGGESTED NOTES AND ACTIVITIES	SUGGESTED RESOURCES
Calculus	• Differentiate functions • integrate functions • solve problems involving 1st order differential equation	• Differentiation • Integration • 1st Order Differential equations	• Differentiating functions • integrating functions • solving problems involving 1st order differential equation	• ICT tools • Braille materials and equipment • Talking books or software • Relevant texts

TOPIC	OBJECTIVES Learners should be able to:	CONTENT: {Skills, Knowledge, Attitudes}	SUGGESTED NOTES AND ACTIVITIES	SUGGESTED RESOURCES
Complex Numbers	• carry out operations with complex numbers • represent complex numbers on an Argand diagram • solve polynomial equations with at least one pair of non-real roots • simplify complex numbers in polar form • illustrate equations and inequalities involving complex numbers by means of loci in an Argand diagram • solve problems related to deMoivre's theorem • solve problems involving complex numbers	• Parts of a complex number • Conjugate, modulus and argument • Operations • Argand diagram • Equations (up to order 5) • Polar form ($r(\cos\theta + i\sin\theta) = re^{i\theta}$) • Loci • deMoivre's Theorem • nth roots of unit	• carrying out operations with complex numbers • representing complex numbers on an Argand diagram • solving polynomial equations with at least one pair of non-real roots • simplifying complex numbers in polar form • illustrating equations and inequalities involving complex numbers by means of loci in an Argand diagram • solving problems related to deMoivre's theorem • solving problems involving complex numbers	• ICT tools • Braille materials and equipment • Talking books or software • Relevant texts
	•	•	•	•

TOPIC	OBJECTIVES Learners should be able to:	CONTENT: {Skills, Knowledge, Attitudes}	SUGGESTED NOTES AND ACTIVITIES	SUGGESTED RESOURCES
Matrices	• carry out basic operations of matrices up to 3x3 • apply the result $(AB)^{-1} = B^{-1}A^{-1}$ for non-singular matrices to solve problems • construct 2 x 2 matrices to represent enlargement, rotation, reflection, stretch and shear transformations • derive the relationship between the area scale-factor of a transformation and the determinant of the corresponding matrix • solve simultaneous equations in 2 or 3 unknowns by reducing them to the matrix equation form ($A\mathbf{X} = \mathbf{b}$)	• Matrices • Mathematical Induction • Groups	• carrying out basic operations of matrices up to 3x3 • applying the result $(AB)^{-1} = B^{-1}A^{-1}$ for non-singular matrices to solve problems • constructing 2 x 2 matrices to represent enlargement, rotation, reflection, stretch and shear transformations • deriving the relationship between the area scale-factor of a transformation and the determinant of the corresponding matrix • solving simultaneous equations in 2 or 3 unknowns by reducing them to the matrix equation form ($A\mathbf{X} = \mathbf{b}$)	• ICT tools • Braille materials and equipment • Talking books or software • Relevant texts

TOPIC	OBJECTIVES Learners should be able to:	CONTENT: {Skills, Knowledge, Attitudes}	SUGGESTED NOTES AND ACTIVITIES	SUGGESTED RESOURCES
Numerical Methods	• distinguish between absolute error and relative error • estimate errors in calculation including the use of $\partial y \approx \frac{dy}{dx}\partial x$ • approximate the root of an equation by graphical means or sign change • derive a - converging iterative - Newton – Raphson formula - Trapezium rule formula for solving a given equation n • solve equations using - iterative procedure - the Newton – Raphson method • recognise cases where the iterative method may fail to converge to the required root • solve problems involving the use of iterative procedures in root finding • solve problems involving the trapezium rule	• Errors • Iterative methods • Newton – Raphson method • Trapezium rule	• distinguishing between absolute error and relative error • estimating errors in calculation including the use of $\partial y \approx \frac{dy}{dx}\partial x$ • approximating the root of an equation by graphical means or sign change • deriving a: - converging iterative - Newton – Raphson - Trapezium rule for solving a given equation • solving equations using - iterative procedure - the Newton – Raphson method • recognising cases where the iterative method may fail to converge to the required root • solving problems involving the use of iterative procedures in root finding • solve problems involving the trapezium rule	• ICT tools • Braille materials and equipment • Talking books or software • Relevant texts

9.0 Assessment

9.1 Assessment Objectives

The assessment will test candidate's ability to: -

9.1.1 use mathematical symbols, terms and definitions appropriately
9.1.2 sketch graphs accurately
9.1.3 use appropriate formulae, algorithms and strategies to solve routine and non-routine problems in Pure Mathematics
9.1.4 solve problems in pure mathematics systematically
9.1.5 apply mathematical reasoning and communicate mathematical ideas clearly
9.1.6 conduct mathematical proofs in the expected manner
9.1.7 construct and use appropriate mathematical models for a given life situation
9.1.8 conduct research projects including those related to enterprise

9.2 Scheme of Assessment

Level 3 Pure Mathematics assessment will be based on 30% continuous assessment and 70% summative assessment.

The syllabus' scheme of assessment caters for all learners. Arrangements, accommodations and modifications must be visible in both continuous and summative assessments to enable candidates with special needs to access assessments and receive accurate performance measurement of their abilities. Access arrangements must neither give these candidates an undue advantage over others nor compromise the standards being assessed.

Candidates who are unable to access the assessments of any component or part of component due to disability (transitory or permanent) may be eligible to receive an award based on the assessment they would have taken.

a) Continuous Assessment

Continuous assessment for Level 3 will consists of topic tasks, written tests, end of term examinations, project and profiling to measure soft skills

i. Topic Tasks

These are activities that teachers use in their day to day teaching. These should include practical activities, assignments and group work activities.

ii. Written Tests

These are tests set by the teacher to assess the concepts covered during a given period of up to a month. The tests should consist of short structured questions as well as long structured questions.

iii. End of term examinations

These are comprehensive tests of the whole term's or year's work. These can be set at school, district or provincial level.

iv. Project

This should be done from term two to term five.

a. Summary of Continuous Assessment Tasks

From term two to five, candidates are expected to have done the following recorded tasks:

- 1 Topic task per term
- 2 Written tests per term
- 1 End of term test per term
- 1 Project in five terms

Detailed Continuous Assessment Tasks Table

Term	Number of Topic Tasks	Number of Written Tests	Number of End of Term Tests	Project	Total
2	1	2	1	1	
3	1	2	1		
4	1	2	1		
5	1	2	1		
Weighting	25%	25%	25%	25%	100%
Actual Weight	7.5%	7.5%	7.5%	7.5%	30%

9.3 Specification Grid for Continuous Assessment

Component Skills	Topic Tasks	Written Tests	End of Term	Project
Skill 1 Knowledge Comprehensive	50%	50%	50%	20%
Skill 2 Application Analysis	40%	40%	40%	40%
Skill 3 Synthesis Evaluation	10%	10%	10%	40%
Total	**100%**	**100%**	**100%**	**100%**
Actual weighting	**7.5%**	**7.5%**	**7.5%**	**7.5%**

b. Summative Assessment

The examination will consist of 2 papers: paper 1 and paper 2, each to be written in 3 hours

The tables below show the information on weighting and types of papers to be offered.

	Paper 1	**Paper 2**	**Total**
Weighting	**35%**	**35%**	**70%**
Type of Paper	Approximately 15 Short answer structured questions, where candidates answer all questions	8 structured questions where candidates answer any 5, and each question carrying 20 marks	
Marks	**100**	**100**	**200**

Specification Grid for Summative Assessment

	Paper 1	**Paper 2**	**Total**	**Weighting**
Skill 1 Knowledge & Comprehension	50%	30%	80%	28%
Skill 2 Application & Analysis	40%	50%	90%	31,5%
Skill 3 Synthesis & Evaluation	10%	20%	30%	10,5%
Total	**100%**	**100%**	**200%**	
Weighting	**35%**	**35%**		**70%**

9.4 Assessment Model

Learners will be assessed using both continuous and summative assessments.